Decision Theory, Problem Set #2

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1. **Preference** Suppose that weak preference ($\succsim$) satisfies the following two constraints.

   Transitivity For all a and b, if $a \succsim b$, and $b \succsim c$, then $a \succsim c$.

   Completeness For all a and b in the domain of $\succsim$, either $f \succsim g$ or $g \succsim f$.

   Suppose, furthermore, that indifference and strong preference are defined in terms of weak preference as follows.

   $a \succ b$ iff $a \succsim b$ and $b \not\succsim a$

   $a \sim b$ iff $a \succsim b$ and $b \succsim a$

   Show that indifference and strong preference have the following properties (for all a and b in the domain of $\succ$ and $\sim$).

   (a) If $a \succ b$ and $b \succ c$, then $a \succ c$
   (b) If $a \succ b$ and $b \sim c$, then $a \succ c$
   (c) If $a \sim b$ and $b \succ c$, then $a \succ c$
   (d) If $a \sim b$ and $b \sim c$, then $a \sim c$
   (e) Exactly one of the following holds: $a \succ b$ or $b \succ a$ or $a \sim b$

2. **Positive Linear Transformations** Where U is an expected utility function that represents a preference ordering $\succsim$, and U^* is an expected utility function, show that

   (a) If there exist some real y and positive real x such that, for every a in the domain of $\succsim$,

   $$U^*(a) = xU^*(a) + y$$

   then U^* represents $\succsim$.

   (b) If there exist no real y and positive real x such that, for every a in the domain of $\succsim$,

   $$U^*(a) = xU^*(a) + y$$

   then U^* does not represent $\succsim$.

3. **Necessary Axioms** Savage showed that every preference relation that satisfies his axioms (which includes the Sure-Thing Principle) can be represented by a probability function and a utility function.

   An axiom is *necessary* (in the context of representation theorems for expected utility theory) if it is satisfied by every set of preferences that can be represented by an expected utility function. Is the Sure-Thing Principle necessary? Why or why not?