Decision Theory, Problem Set #1*

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July 28, 2014

1. **Train or Plane?**

   As far as cost and safety are concerned, train and plane provide equally good ways of getting from Las Pulgas to San Francisco. However, the trip takes 8 hours by train, and 3 hours by plane, unless the San Francisco airport proves to be fogged in, in which case the plane trip takes 15 hours. According to the weather forecast, there are 7 chances out of 10 that San Francisco will be fogged in. Use expected utility theory to decide whether to take the train or plane. (Assume that travel does *not* have diminishing marginal utility, so that each additional hour of travel lowers the utility of the outcome by the same amount.)

2. **The Point of Balance** In problem 1, what must the probability of fog be, if plane and train are to be equally good choices?

3. **Beyond Matrices** Many problems are simplified if we use different sets of relevant conditions for different acts, in a way that is impossible in the usual matrix formulation. Example: the variant of problem 1 in which the conditions relevant to going by plane are still fog or not (at the airport) but the conditions relevant to going by train are quite different: snow or not (in a mountain pass). For the act of going by plane, the relevant events (with their probabilities) and outcomes (with their utilities) are given by the following matrix:

	fog ($P = 0.7$)	**no fog** ($P = 0.3$)
plane	$u = -15$	$u = -3$

   (The expected utility of going by plane is $0.7 \times -15 + 0.3 \times -3 = -11.4$.)

   But if the probability of snow in the pass is 0.5, and the train trip takes 10 hours or 8, depending on whether there is snow in the pass, the relevant information about going by train would be shown in a row of a different matrix, with different column headings:

[0] Problems lifted nearly verbatim from Richard Jeffrey's *The Logic of Decision*, 2nd edition, 1989. Minor alterations to adjust for some of my presentation choices in the lecture.

	snow ($P = 0.5$)	**no snow** ($P = 0.5$)
train	$u = -10$	$u = -8$

(The expected utility of going by plane is $0.5 \times -10 + 0.5 \times -8 = -19$.)

Then, in order to see that going by train is preferable, there is no need to use the standard matrix format, as below. Problem: discuss (and, if possible, overcome) the difficulties of filling in the blanks of the standard matrix with probabilities and utilities on the basis of the information given so far.

	fog snow	**fog no snow**	**no fog snow**	**no fog no snow**
train				
plane				

4. **Pascal's Wager** The agent is trying to decide whether to undertake a course of action designed to overcome his intellectual scruples and lead to belief in God. The possible states of the world and their probabilities, together with the available acts, their outcomes, and the outcome utilities, are given by the following matrix. x, y, and z are finite.

	God exists ($P = 0.000001$)	**there is no God** ($P = 0.999999$)
succeed in believing	eternal life ($u = \infty$)	finite life, deluded ($u = x$)
remain an atheist	a bad situation ($u = y$)	status quo ($u = z$)

What should he do?

5. **Fermat's Wager** The terms "Fermat's last theorem" and "the Fermat conjecture" are applied to the following assertion (for which Pierre de Fermat claimed to have found a marvellous proof, which was unfortunately too long to fit in the margin where the assertion itself was found, in Fermat's handwriting, after his death).

If x, y and z are positive integers and $x^n + y^n = z^n$, then n is 1 or 2.

It took 358 years for someone to produce a proof. (Andrew Wiles finally proved the theorem in 1995.) Now suppose (quite implausibly) that Fermat knew he had no proof, but simply wanted to enhance his posthumous reputation as a mathematician. His problem, then, was to choose one of three acts: assert the conjecture, deny it, or leave the margin blank. If Fermat was quite confident that a proof or counterexample would be found, then the relevant states might be: that the conjecture is true, and that it is false. It is plausible (why?) to suppose that the problem is correctly described by the following matrix.

	true ($P = x$)	**false** ($P = 1 - x$)
assert	$u = 1/x$	$u = -1$
deny	$u = -1$	$u = 1/(1-x)$
leave blank	$u = 0$	$u = 0$

Knowing that Fermat asserted his conjecture, what can we conclude about the probability x that he attributed to it?

6. **The St. Petersburg Game** A fair coin is tossed repeatedly until the first tails turns up (on toss number n, say) at which point the player receives 2^n ducats. What is the expected monetary value of this game?